Quadratic Equations

Question1

With usual notations in $\triangle \mathrm{ABC}$, if $\angle \mathrm{B}=\frac{\pi}{2}$, and $\tan \frac{\mathrm{A}}{2}, \tan \frac{\mathrm{C}}{2}$ are roots of equation $\mathrm{p} x^2+\mathrm{q} x+\mathrm{r}=0, \mathrm{p} \neq 0$, then MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\mathrm{p}+\mathrm{q}=\mathrm{r}$

B. $r+p=q$

C. $r=p$

D. $\mathrm{p}=\mathrm{q}$

Answer: A

Solution:

Given

- $\angle B=90^{\circ} \Rightarrow \dfrac{B}{2}=45^{\circ}$ so $\tan \frac{B}{2}=1$.
- Roots of $p x^2+q x+r=0$ are $\alpha=\tan \frac{A}{2},\ \beta=\tan \frac{C}{2}$.

Formulae Used

1. For quadratic $p x^2+q x+r=0$:
   $\alpha+\beta=-\dfrac{q}{p}, \quad \alpha \beta=\dfrac{r}{p}$.
2. Half-angle identity in any triangle:

$$\tan \frac{A}{2} \tan \frac{B}{2}+\tan \frac{B}{2} \tan \frac{C}{2}+\tan \frac{C}{2} \tan \frac{A}{2}=1$$

Steps

Since $\tan \frac{B}{2}=1$,

$$\alpha \cdot 1+1 \cdot \beta+\alpha \beta=1 \Rightarrow(\alpha+\beta)+\alpha \beta=1.$$

Substitute from (1):

$$\left(-\frac{q}{p}\right)+\frac{r}{p}=1 \Rightarrow \frac{r-q}{p}=1 \Rightarrow r-q=p \Rightarrow \boxed{p+q=r}.$$

Concept Snapshot

Use the standard identity for $\tan$ of half-angles in a triangle and the sum/product of roots of a quadratic to relate p, q, r.

✅ **Final Answer**

(A) $p+q=r$

Question2

If $x=-2+\sqrt{-3}$, then the value of $2x^4+5x^3+7x^2-x+38$ is equal to MHT CET 2025 (23 Apr Shift 1)

Options:

A. 1

B. -2

C. 3

D. 5

Answer: C

Solution:

Given: $x=-2+\sqrt{-3}=-2+i\sqrt{3} \Rightarrow$ conjugate exists ⇒
x satisfies $(x+2)^2+3=0 \Rightarrow x^2+4x+7=0.$

Reduce powers (mod x^2+4x+7):
$x^2=-4x-7$
$x^3=x(-4x-7)=-4x^2-7x=9x+28$
$x^4=x(9x+28)=9x^2+28x=-36x-63+28x=-8x-63$

Evaluate:

$$2x^4+5x^3+7x^2-x+38=2(-8x-63)+5(9x+28)+7(-4x-7)-x+38$$

$$= (-16x - 126) + (45x + 140) + (-28x - 49) - x + 38$$
$$= 0x + (-126 + 140 - 49 + 38) = 3.$$

✅ Final Answer: 3 (Option C)

Question3

If $\mathrm{f}(x) = 2x^3 + \mathrm{m}x^2 - 13x + \mathrm{n}$ and 2,3 are the roots of the equation $\mathrm{f}(x) = 0$ then the value of $4\,\mathrm{m} + 5\mathrm{n}$ is MHT CET 2025 (20 Apr Shift 2)

Options:

A. 30

B. 100

C. 130

D. 150

Answer: C

Solution:

Given: $f(x) = 2x^3 + mx^2 - 13x + n$, roots $2, 3$.

- $f(2) = 0 \Rightarrow 16 + 4m - 26 + n = 0 \Rightarrow 4m + n = 10$
- $f(3) = 0 \Rightarrow 54 + 9m - 39 + n = 0 \Rightarrow 9m + n = -15$

Subtract: $(9m + n) - (4m + n) = -15 - 10 \Rightarrow 5m = -25 \Rightarrow m = -5$.
Then $n = 10 - 4(-5) = 30$.

$$4m + 5n = 4(-5) + 5(30) = -20 + 150 = 130$$

✅ Final Answer: 130 (Option C)

Question4

If triangle ABC is a right angled at A and $\tan\frac{B}{2}, \tan\frac{C}{2}$ are roots of the equation $ax^2+bx+c=0, a\neq 0$, then MHT CET 2025 (19 Apr Shift 2)

Options:

A. $a+c=b$

B. $a+b=c$

C. $\mathrm{b}+\mathrm{c}=a$

D. $a+c=2b$

Answer: B

Solution:

Given: Roots = $\tan\frac{B}{2}, \tan\frac{C}{2}$.
From quadratic: $\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$.

Since $A=90^\circ$, $\tan\frac{A}{2}=1$. Identity:

$$\alpha+\beta+\alpha\beta=1$$

So,

$$-\frac{b}{a}+\frac{c}{a}=1 \Rightarrow c-b=a \Rightarrow a+b=c$$

✅ Final Answer: (B) $a+b=c$

Question5

The value of ' a ' so that the sum of squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assumes the least value is MHT CET 2025 (19 Apr Shift 2)

Options:

A. 2

B. 1

C. 4

D. 0

Answer: B

Solution:

Equation given

$$x^2-(a-2)x-a+1=0$$

Step 1: Sum and product of roots

If roots = α, β:

- Sum: $\alpha+\beta=a-2$
- Product: $\alpha\beta=-a+1$

Step 2: Sum of squares

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(a-2)^2-2(-a+1)$$

$$=(a^2-4a+4)+2a-2=a^2-2a+2$$

Step 3: Minimize

Expression: a^2-2a+2.

Complete square:

$$a^2 - 2a + 2 = (a-1)^2 + 1 \quad \geq 1$$

Minimum value = 1, at $a = 1$.

✅ Final Answer:

Option (B): 1

Question6

The Number of values of C that satisfy the conclusion of Rolle's theorem in case of following function $\mathrm{f}(x) = \sin 2\pi x, x \in [-1, 1]$ is MHT CET 2024 (15 May Shift 2)

Options:

A. 02

B. 04

C. 03

D. zero

Answer: B

Solution:

$$\mathrm{f}(x) = \sin 2\pi x$$

$$\therefore \quad \mathrm{f}'(x) = 2\pi \cos 2\pi x$$

$$\text{Now, } \mathrm{f}'(\mathrm{C}) = 0$$

$$\Rightarrow \quad 2\pi \cos 2\pi \mathrm{C} = 0$$

$$\Rightarrow \cos 2\pi \mathrm{C} = 0$$

$$\Rightarrow 2\pi C = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\ldots [\because x \in (-1, 1) \Rightarrow 2\pi x \in (-2\pi, 2\pi)]$$

$$\Rightarrow C = \frac{-3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}$$

Number of values of C $= 4$

Question7

The equation $e^{\sin x} - e^{-\sin x} = 4$ has ________ solutions. MHT CET 2024 (15 May Shift 1)

Options:

A. 2

B. 4

C. 3

D. no

Answer: D

Question8

In a $\triangle PQR, m\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0 (a \neq 0)$, then MHT CET 2024 (11 May Shift 2)

Options:

A. a + b = c

B. b + c = a

C. a + c = b

D. b = c

Answer: A

Solution:

In $\triangle PQR$

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\therefore \quad \angle P + \angle Q + \frac{\pi}{2} = 180^\circ$$

$$\therefore \quad \angle P + \angle Q = \frac{\pi}{2}$$

$$\therefore \quad \frac{\angle P}{2} + \frac{\angle Q}{2} = \frac{\pi}{4}$$

$\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of the equation $ax^2 + bx + c = 0$...[Given]$\therefore$ Sum of roots $= \frac{-b}{a} \tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = \frac{-b}{a}$ Also,

$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$ Using, $\tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\tan\frac{Q}{2}}$, we get

$$\Rightarrow \tan\left(\frac{\pi}{4}\right) = \frac{\frac{-b}{a}}{1 - \frac{c}{a}}$$

$$\Rightarrow 1 = \frac{-b}{a - c}$$

$$\Rightarrow a - c = -b$$

$$\Rightarrow a + b = c$$

Question9

Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - \mathrm{q}x + \mathrm{r} = 0$. Then the value of r is MHT CET 2024 (09 May Shift 2)

Options:

A. $\frac{2}{9}(p-q)(2q-p)$

B. $\frac{2}{9}(q-p)(2p-q)$

C. $\frac{2}{9}(q-2p)(2q-p)$

D. $\frac{2}{9}(2p-q)(2q-p)$

Answer: D

Solution:

α and β are the roots of $x^2 - \mathrm{p}x + \mathrm{r} = 0$

$\therefore$ sum of roots $= \alpha + \beta$

$\Rightarrow \alpha + \beta = p \ldots (i)$

$\frac{\alpha}{2}, 2\beta$ are the roots of $x^2 - \mathrm{q}x + \mathrm{r} = 0$

$\therefore$ sum of roots $= \mathrm{q}$

$$\Rightarrow \frac{\alpha}{2} + 2\beta = q$$
$$\Rightarrow \alpha + 4\beta = 2q \ldots (ii)$$

Subtracting (i) from (ii), we get

$$3\beta = 2q - p$$
$$\Rightarrow \beta = \frac{2q-p}{3}$$

From (i),

$$\alpha + \frac{2q-p}{3} = p$$
$$\Rightarrow \alpha = \frac{2(2p-q)}{3}$$

Product of roots $= \alpha\beta$

$$\Rightarrow r = \alpha\beta = \frac{2(2p-q)}{3} \times \frac{2q-p}{3}$$
$$= \frac{2}{9}(2p-q)(2q-p)$$

Question10

The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x, has real roots. Then p can take any value in the interval MHT CET 2024 (03 May Shift 2)

Options:

A.

$(0, 2\pi)$

B.

$(-\pi, 0)$

C.

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D.

$(0, \pi)$

Answer: D

Solution:

Given equation is $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$
Comparing with $ax^2 + bx + c = 0$, we get $a = \cos p - 1, b = \cos p, c = \sin p$

It has real roots. $\therefore \quad b^2 - 4ac \geq 0$

$$\Rightarrow \cos^2 \mathrm{p} - 4(\cos \mathrm{p} - 1)(\sin \mathrm{p}) \geq 0$$
$$\Rightarrow \cos^2 \mathrm{p} - 4 \sin \mathrm{p} \cos \mathrm{p} + 4 \sin \mathrm{p} \geq 0$$
$$\Rightarrow \cos^2 \mathrm{p} - 4 \sin \mathrm{p} \cos \mathrm{p} + 4 \sin^2 \mathrm{p} + 4 \sin \mathrm{p} - 4 \sin^2 \mathrm{p} \geq 0$$

$$\Rightarrow (\cos p - 2 \sin p)^2 + 4 \sin p(1 - \sin p) \geq 0$$

$\therefore \quad (\cos p - 2 \sin p)$ is always positive

$\therefore \quad 1 - \sin p \geq 0$ for all values of $p, p \in (0, \pi)$

Question11

Let α and β be two real roots of the equation $(\mathrm{k}+1) \tan^2 x - \sqrt{2} \lambda \tan x = (1 - \mathrm{k})$ where $\mathrm{k}(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is MHT CET 2024 (03 May Shift 2)

Options:

A.

$5\sqrt{2}$

B.

$10\sqrt{2}$

C.

10

D.

5

Answer: C

Solution:

$$(\mathrm{k}+1)\tan^2 x-\sqrt{2}\lambda\tan x=(1-\mathrm{k})\quad \ldots(i)$$
$$\Rightarrow(\mathrm{k}+1)\tan^2 x-\sqrt{2}\lambda\tan x+(\mathrm{k}-1)=0\quad \ldots(ii)$$

α and β are two real roots.

$$\therefore\quad \tan\alpha+\tan\beta=\frac{\sqrt{2}\lambda}{k+1}$$
$$\tan\alpha\cdot\tan\beta=\frac{k-1}{k+1}$$

Now, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{\sqrt{2}\lambda}{\mathrm{k}+1}}{1-\frac{(\mathrm{k}-1)}{\mathrm{k}+1}}$$
$$=\frac{\sqrt{2}\lambda}{2}$$

$$\Rightarrow\tan(\alpha+\beta)=\frac{\lambda}{\sqrt{2}}$$
$$\Rightarrow\tan^2(\alpha+\beta)=\frac{\lambda^2}{2}$$
$$\Rightarrow 50=\frac{\lambda^2}{2}$$
$$\Rightarrow\lambda^2=100$$
$$\Rightarrow\lambda=10$$

Question12

The number of roots of the equation, $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$ in the interval $[0,\pi]$, is equal to MHT CET 2024 (03 May Shift 1)

Options:

A.

4

B.

8

C.

3

D.

2

Answer: A

Solution:

$(81)^{\sin^2 x}+(81)^{\cos^2 x}=30\ldots(i)$

Let $y=81^{\sin^2 x}$

$\therefore\quad 81^{\cos^2 x}=81^{(1-\sin^2 x)}=\frac{81}{81^{\sin^2 x}}=\frac{81}{y}$ $\therefore$ Equation (i) becomes $y+\frac{81}{y}=30$

$\therefore\quad y^2-30y+81=0$

$\therefore\quad (y-27)(y-3)=0$

$\therefore\quad y=27$ or 3

$\therefore\quad 81^{\sin^2 x}=27\quad$ or $\quad 81^{\sin^2 x}=3$

$\therefore\quad 3^{4\sin^2 x}=3^3\quad$ or $\quad 3^{4\sin^2 x}=3^1$

$\therefore\quad 4\sin^2 x=3\quad$ or $\quad 4\sin^2 x=1$ $\therefore\quad \sin x=\frac{\sqrt{3}}{2},\frac{1}{2}\ldots[\because x\in[0,\pi]]$ $\therefore\quad x=\frac{\pi}{3},\frac{2\pi}{3},\frac{\pi}{6},\frac{5\pi}{6}\ldots[\because x\in[0,\pi]]$ $\therefore$ Required number of roots are 4 .

Question13

The equation $x^3+x-1=0$ has MHT CET 2023 (14 May Shift 1)

Options:

A.

no real root.

B.

exactly two real roots.

C.

exactly one real root.

D.

more than two real roots.

Answer: C

Solution:

Let $f(x) = x^3 + x - 1$

A root of $f(x)$ exists, if $f(x) = 0$ for at least one value of x.

$$\begin{aligned} f(0) &= -1 < 0 \\ f(1) &= 1 > 0 \end{aligned}$$

$\therefore$ By intermediate value theorem, there has to be a point 'c' between 0 and 1 such that $f(x) = 0$.

$\therefore$ The given equation has exactly one real root.

Alternate Method:

Let $f(x) = x^3 + x - 1$

$$\begin{aligned} \therefore \quad & f'(x) = 3x^2 + 1 \\ & \Rightarrow f'(x) > 0 \quad \forall x \in R \end{aligned}$$

$\Rightarrow f(x)$ is an increasing function.

$\Rightarrow f(x)$ intersects X-axis at only one point.

$\therefore$ The given equation has exactly one real root.

Question14

In $\triangle ABC$, with tustal notations, $m\angle C = \frac{\pi}{2}$, if $\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are the roots of the equation. $a_1x^2 + b_1x + c_1 = 0\ (a_1 \neq 0)$, then MHT CET 2023 (09 May Shift 2)

Options:

A.

$a_1 + b_1 = c_1$

B.

$b_1 + c_1 = a_1$

C.

$a_1 + c_1 - b_1$

D.

$b_1 = c_1$

Answer: A

Solution:

$$\begin{aligned} & \text{In } \triangle \mathrm{ABC}, \\ & \angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ \\ \therefore \quad & \angle \mathrm{A} + \frac{\pi}{2} + \angle \mathrm{B} = 180^\circ \\ \therefore \quad & \angle \mathrm{A} + \angle \mathrm{B} = \frac{\pi}{2} \\ \therefore \quad & \frac{\angle \mathrm{A}}{2} + \frac{\angle \mathrm{B}}{2} = \frac{\pi}{4} \end{aligned}$$

$\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are roots of equation

$\mathrm{a}_1 x^2 + \mathrm{b}_1 x + \mathrm{c}_1 = 0 \ldots [Given]$

$$\begin{aligned} \therefore \quad \text{Sum of roots} &= \frac{-b_1}{a_1} \\ \tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right) &= \frac{-b_1}{a_1} \end{aligned}$$

Also, $\tan\left(\frac{\mathrm{A}}{2}\right) \cdot \tan\left(\frac{\mathrm{B}}{2}\right) = \frac{\mathrm{c}_1}{\mathrm{a}_1}$ Using $\tan\left(\frac{\mathrm{A}}{2} + \frac{\mathrm{B}}{2}\right) = \frac{\tan\frac{\mathrm{A}}{2} + \tan\frac{\mathrm{B}}{2}}{1 - \tan\frac{\mathrm{A}}{2}\tan\frac{\mathrm{B}}{2}}$, we get

$$\begin{aligned} \tan\left(\frac{\pi}{4}\right) &= \frac{\frac{-b_1}{a_1}}{1 - \frac{c_1}{a_1}} \\ 1 &= \frac{-b_1}{a_1 - c_1} \\ a_1 - c_1 &= -b_1 \\ a_1 + b_1 &= c_1 \end{aligned}$$

Question15

If $f(x) = ax^2 + bx + 2$ and $f(1) = 4, f(3) = 38$, then $a - b =$ MHT CET 2020 (20 Oct Shift 2)

Options:

A.

15

B.

-2

C.

2

D.

8

Answer: D

Solution:

$$
\begin{aligned}
& f(1)=4; \quad a+b=2\ldots(1) \\
& f(3)=38; \quad 9a+3b+2=38- \ (2) \\
& 3a+b=12
\end{aligned}
$$

Solving (1) \& (2)

$$
\begin{aligned}
& a=5, b=-3 \\
& a-b=8
\end{aligned}
$$

Question16

The quadratic equation whose roots are the numbers having arithmetic mean 34 and geometric mean 16 is MHT CET 2020 (19 Oct Shift 2)

Options:

A.

$x^2+68x-256=0$

B.

$x^2-68x-256=0$

C.

$x^2-68x+256=0$

D.

$x^2+68x+256=0$

Answer: C

Solution:

Quad Eq^n :→

AM of $a, b = 34$

$a + b = 68$

GM of $a, b, = \pm 6$

$a, b \rightarrow$ roots $\rightarrow$ $ab = 256$
$x^2 - 68x + 256 = 0$

Question17

If the A.M. and G.M. of the roots of a quadratic equation in x are p and q respectively, then its equation is MHT CET 2020 (15 Oct Shift 2)

Options:

A.

$x^2 + 2px + q^2 = 0$

B.

$x^2 + px + q^2 = 0$

C.

$x^2 - px + q^2 = 0$

D.

$x^2 - 2px + q^2 = 0$

Answer: D

Solution:

$v' = 2v$

$\frac{x_1+x_2}{2} = p \quad \& \quad \sqrt{x_1 x_2} = q$

None $ax^2 + bx + c = 0$

$b = -(x_1 + x_2)$

$c = x_1 x_2$

$b = -2p$

$\therefore rq^n = x^2 - 2px + q^2 = 0$

Question18

If m_1 and m_2 are slopes of the lines represented by $\left(\sec^2\theta - \sin^2\theta\right)x^2 - 2\tan\theta xy + \sin^2\theta y^2 = 0$, then $|m_1 - m_2| =$
MHT CET 2020 (14 Oct Shift 1)

Options:

A.

1

B.

2

C.

4

D.

3

Answer: B

Solution:

$\left(\sec^2\theta - \sin^2\theta\right)x^2 - 2\tan\theta xy + \sin^2\theta y^2 = 0$

$\sqrt{ax^2 + 2bxy + by^2} = 0$

$|m_1 - m_2| = ?$

$|m_1 - m_2| = \sqrt{\left(m_1 + m_2\right)^2 - 4m_1m_2}$

$m_1 + m_2 = \frac{-2h}{b}$

$m_1m_2 = \frac{a}{b}$

$m_1 - m_2 | = \sqrt{\frac{4\tan^2\theta - 4\left(\sec^2\theta - \sin^2\theta\right)}{\sin^2\theta}}$

$= 2$

Question19

If $A = \left\{x \in \frac{R}{x^2} + 5|x| + 6 = 0\right\}$ then $n(A) =$ **MHT CET 2019 (Shift 1)**

Options:

A.

0

B.

4

C.

1

D.

2

Answer: A

Solution:

We have, $A = \left\{x \in \frac{R}{x^2} + 5|x| + 6 = 0\right\}$

Now, $x^2 + 5|x| + 6 = 0 \quad \ldots(i)$

$\Rightarrow x^2 + 3|x| + 2|x| + 6 = 0$

$\Rightarrow |x|(|x| + 3) + 2(|x| + 3) = 0$

$\Rightarrow (|x| + 2)(|x| + 3) = 0$

$\Rightarrow |x| = -2 |x| = -3$

$\Rightarrow x \neq 2 \, or \, x \neq 3$

Here, no value of x satisfy the Eq. (i).

$\therefore n(A) = 0$

Question20

If α and β are roots of the equation $x^2 + 5|x| - 6 = 0$ then the value of $|\tan^{-1}\alpha - \tan^{-1}\beta|$ is **MHT CET 2017**

Options:

A. $\frac{\pi}{2}$

B. 0

C. π

D. $\frac{\pi}{4}$

Answer: A

Solution:

$x^2 + 5|x| - 6 = 0$

$|x|^2 + 5|x| - 6 = 0$

$|x|^2 + 6|x| - |x| - 6 = 0$

$|x|(|x| + 6) - 1(|x| + 6) = 0$

$(|x| - 1)(|x| + 6) = 0$

$|x| = 1, \text{nbsp;} |x| \neq -6$ (Since modulus can not be giving negative values)

$\therefore \quad |x| = 1$

$\therefore \quad x = \pm 1$

$\alpha = 1, \beta = -1$

$\therefore \quad |\tan^{-1}\alpha - \tan^{-1}\beta| = |\tan^{-1}1 - \tan^{-1}(-1)|$

$= |\frac{\pi}{4} - (-\frac{\pi}{4})|$

$= |\frac{\pi}{2}| = \frac{\pi}{2}$
